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Oh, the yogurt you'll make:

I once worked in a small yogurt shop in North Salt Lake called, "Yogi Berry." While working there I sold frozen yogurt to sweet-toothed locals quite frequently. We had six different types of frozen yogurt at any given time (although they were regularly swapped out) and forty toppings from which one might choose. As I was working one day I began to wonder how many different combinations were possible to make from such a collection of yogurt and sprinkles and such. From this idea grew my desire to research and learn how to find the answer, besides, I really wanted to impress my boss.

Situation:

I need to find out how many possible combinations of frozen yogurt I might be able to make by selecting anywhere from one to six types of yogurt at a time. I then need to mathematically calculate how many different types of toppings I could combine with my yogurt to make the treat complete. Using the Combinations formula found in chapter 7-E, I found that I was able to make **63** different combinations of frozen yogurt (Bennett).

$$_6C_1 = \frac{6!}{5!\,1!} = 6$$

$$_6C_2 = \frac{6!}{4!\,2!} = 15$$

$$_6C_3 = \frac{6!}{3!\,3!} = 20$$

$$_6C_4 = \frac{6!}{2!\,4!} = 15$$

$$_6C_5 = \frac{6!}{1!\,5!} = 6$$

$$_6C_6 = \frac{6!}{0!\,6!} = 1$$

Total: 63

In order to find the different combinations of frozen yogurt and toppings more easily one might use elements and strategies found in Pascal's triangle. Although Blaise Pascal, the French mathematician the triangle is named after, didn't invent the triangle, he did revolutionize the way that it was applied and the usefulness of the correlations among the many numbers. In his work Pascal wrote about the arithmetic of triangles. In 1654 he looked at a correlation that had been researched by the ancient Chinese, Greek, and Indian people that concerned triangles.

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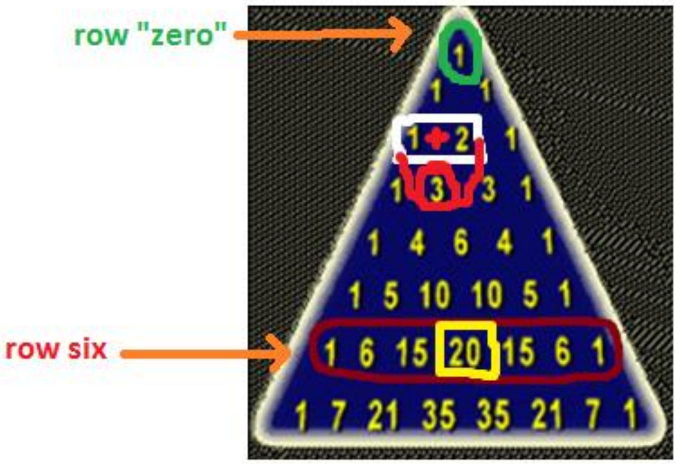
This is Pascal's triangle:



(Figure 1)

The *first* row (from left to right) has only one "1" is called row "zero." Each row down from that is the first, second, and third… respectively. This triangle is formed by adding two numbers side by side on one column and putting that sum directly below those two numbers as is represented by the white box and red circle around the number 3 in figure 1. This is one of the patterns present in Pascal's triangle. If you'll notice in figure 1 on the sixth row that has a convenient red line around it, we have found the exact numbers of combinations as we did by taking the longer method of using the combinations formula.

As an example of the simplicity and usefulness of this triangle we will take the problem of figuring out how many combinations are possible with six (6) yogurts choosing three (3) at a time. We can go down to the 6^{th} row and go over to the third number (this has a yellow square around it). We skip the first (1) in the sequence as we did while counting the rows and call that column "zero." By locating the sixth row and going to the third column over we find that there are **twenty** combinations possible. This is precisely what we found by doing it the long way:

$$_6C_3 = \frac{6!}{3!\,3!} = 20$$

One helpful expression that Pascal realized is: $\mathbf{2^n}$. One can supplant the superscript of **n** for any row on the triangle and come up with the number of combinations. By using the example of having six yogurts one can easily find that $\mathbf{2^6 = 64}$. This accounts for the possibility of not having any yogurt at all, but for all practical purposes, let's suppose that if a person goes into the shop, they're going to buy a yogurt. From the **64** we will subtract **1** and come to the conclusion that we have **63** different combinations of yogurt possible. $\mathbf{2^6-1=63}$ Wasn't that easier?

Now, we can use this same principle in order to find out how many different types of toppings are possible: $\mathbf{2^{40} = 1{,}099{,}511{,}627{,}776}$. We do want to account for the fact that it is possible that one could come in the shop and get frozen yogurt with no toppings, so we will not subtract **1.**

With that information we can now see that:

63 yogurt combinations X **1,099,511,627,776** topping combinations **=**

$\mathbf{6.9 \times 10^{13}}$ total combinations

OR…

69,269,232,549,888 TOTAL COMBINATIONS!!

So, the next time I walk into Yogi Berry I will realize that I will have at least 63 yogurt combinations I can create. I can't complain about anything other than having so many choices. I'll also be able to impress my former boss because I'll know how many combinations I could have. In conclusion, I found that even with a relatively small amount of yogurts and a bundle of toppings I can be creative and rest assured that it will be highly unlikely that I'll ever make the same yogurt twice. I'm free to be inventive and come up with whatever wild and crazy concoctions I can think up.

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